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On Boundary Conditions for a Simulation Plasma in a Magnetic Field

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Various methods of treating particles at the boundary walls in a magnetized simulation plasma are studied extensively. Especially, Lee and Okuda's 'reflection scheme' and the numerical instability caused by this scheme alone are investigated in detail. Methods (Method I, Method II, and the random reflection method), which eliminate the numerical instability, are proposed. They are simple and straightforward, and introduce neither a density gradient nor a surface current next to the walls. Lee and Okuda's complete method is stable by virtue of the introduction of a 'smoothing scheme,' and our methods provide an alternative.

I. INTRODUCTION

Particle simulation has contributed to the understandings of plasma physics. In particular it is a powerful tool for understanding transport phenomena across a magnetic field. Plasma diffusion due to low frequency convective cells [1] and electron diffusion caused by low frequency ion fluctuations or lower hybrid waves [2] are some examples.

In many of these, the infinite and homogeneous plasma is simulated by using periodic boundary conditions. There are no numerical difficulties with these boundary conditions. However all experimental plasmas are finite and bounded, and hence inhomogeneous. Inhomogeneity in plasmas produces many interesting phenomena, for example drift-wave instabilities and trapped-particle instabilities. It is important to use a bounded plasma model to simulate a inhomogeneous plasma. Moreover

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bounded plasma model itself involves many significant phenomena; Gould Trivelpiece waves [3], sheath layers and their oscillations [4], etc.

Boundary conditions for a finite simulation plasma have been studied in some papers [5–7]. Due to the restrictions on memory and *CPU* time for computers, we can only simulate a small volume of plasma compared with real one. Further we must determine the boundary conditions in such a way so as not to disturb the phenomena of interest in the interior of the plasma. If one is not careful, the boundary phenomenon can dominate the whole plasma behaviour.

The important boundary condition for simulating a bounded plasma in a static external magnetic field is the treatment of particles at plane walls which are parallel to the magnetic field. These walls modify the cyclotron motion of charged particles and cause shifts of guiding centers of particles which hit the walls. Then macroscopic density gradients, currents near the boundaries and surface waves can be produced. These effects may seriously obscure the phenomena of interest in a simulation plasma.

W. W. Lee and H. Okuda [5] examined the above mentioned boundary effects and proposed a rather special "reflection scheme" of particles in which the positions of the guiding centers of the particles striking the wall do not suffer any shift along the wall. This "reflection scheme" alone has generated the numerical instability near the boundary [5]. Therefore the "smoothing scheme" has been necessitated to smooth out the potential fluctuations associated with the instability [5]. Lee and Okuda's method (LOM) consists of these two schemes and has been used with satisfactory results in simulating drift wave instabilities [5, 8, 9, 10].

Although LOM is useful to simulate a bounded plasma, it includes a nonphysical effect; a macroscopic surface current layer exists due to the counterrotating particles (while the surface instability introduced by this current is remedied by the "smoothing scheme," the phenomenon next to the wall is still nonphysical). In this paper we present methods which exclude such a spurious effect. The methods are simple relative to LOM. They are stable without the "smoothing scheme" with no mathematical or physical complications. We can use these methods even in magnetostatic and electromagnetic codes because of having no artificial current.

In Sec. 2, the "reflection scheme" of Lee and Okuda is studied in detail. The study is instructive to know the importance and the subtlety of the boundary problem because the instability caused by using this scheme alone is typical of the boundary phenomena. The study also improves our physical understandings of numerical products next to the wall. In Sec. 3, simple methods which involve no spurious nonphysical effect are presented as well as comparisons with other existing methods. Concluding remarks and discussions are given in Sec. 4.

II. Lee and Okuda's Method

First we review the method of treating particles at the boundary proposed by W. W. Lee and H. Okuda [5]. For simplicity let us consider a two dimensional system which is normal to the magnetic field in the z direction; the particle motions along the magnetic field are neglected; the plasma is filled in the positive x region bounded by a wall at $x = 0$. We concentrate our attention on this wall. Further let us consider the homogeneous plasma next to the wall because any successful boundary condition should be quiet and stable at the least in this situation.

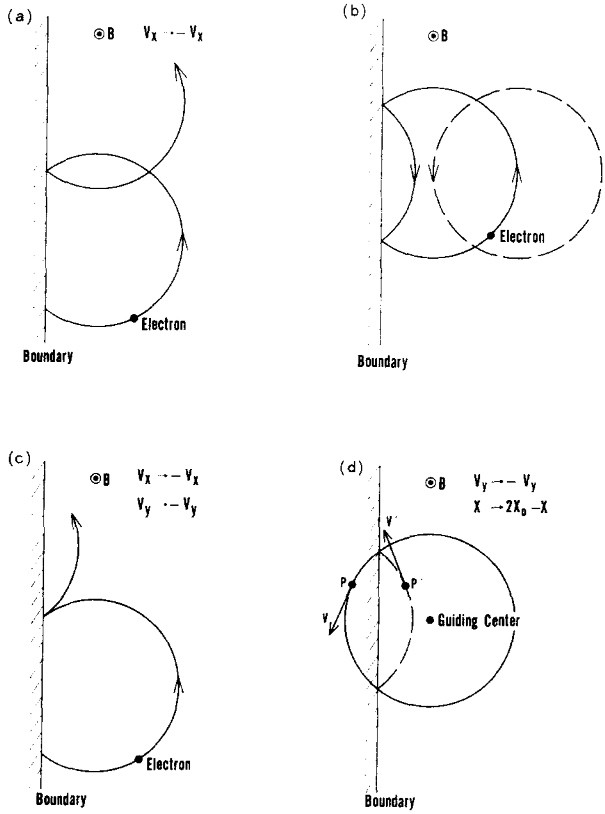

FIG. 1. Illustrations for different boundary conditions. Reflecting boundary condition is shown in (a). Lee and Okuda's 'reflection method' is shown in (b). The origin of the surface current in this method is also shown. Method I is shown in (c). Initial condition in Method II is shown in (d) where x_0 indicates the position of the wall. It shows the initial treatment of the particles which are outside the system due to the uniform loading of the guiding centers. The combination of this initial condition and the reflecting boundary condition is Method II. Note that this initial condition is the one that should be used with Method I.

Lee and Okuda claim that the *reflecting boundary condition*, where a particle exiting from the system is reflected back into the original system at every time step with $x \to -x$ and $v_x \to -v_x$ (see Fig. 1a), causes two effects; (1) a sheath current along the wall and (2) a sharp density gradient near the boundary the width of which is of the order of a Larmor radius. These effects come from the disturbance of the guiding-center positions of the particles which strike the wall. Then, they proposed another method of reflection which is illustrated in Fig. 1b. In their method, particles which hit the wall are reflected by the *reflecting boundary condition* but moved in a inversed magnetic field until they hit the wall again where they are reflected and moved in a normal magnetic field. It is to be noted that both the particles which move in a normal magnetic field and ones which move in a inversed magnetic field, feel the electric fields in their actual positions without changing the signs of their charges. Hence total energy is conserved as well as the total charge. The guiding centers of the particles change in the x direction each time they hit the wall. However they do not suffer any shift along the wall.

Lee and Okuda's method (LOM) consists of this "reflection scheme" and the "smoothing scheme" as stated in the introductory section. In this section, we only consider the "reflection scheme" and the results caused by using this scheme alone, because typical boundary phenomena are clearly seen for this case. The study gives us good understanding of the boundary phenomena as well as the suggestion to the simpler methods being stable without the "smoothing scheme."

The merit of LOM is that it produces no density gradient near the wall. However, it causes a macroscopic current in the y direction. In LOM, we have the macroscopic density $n_\sigma^{\mathrm{LOM}}(x)$ and velocity $\mathbf{v}_\sigma^{\mathrm{LOM}}(x)$ in the following forms;

$$n_\sigma^{\mathrm{LOM}}(x) = \text{const.}, \tag{1a}$$

$$\mathbf{v}_\sigma^{\mathrm{LOM}}(x) = \operatorname{sgn}(q_\sigma) \sqrt{\frac{2}{\pi}}\, v_{t\sigma} \exp\left[-\frac{x^2}{2\rho_\sigma^{\,2}}\right] \mathbf{i}_y , \tag{1b}$$

where q_σ, $v_{t\sigma}$ and ρ_σ are the charge, the thermal velocity $(=(T_\sigma/m_\sigma)^{1/2})$ and the Larmor radius $(=v_{t\sigma}/\omega_{c\sigma})$ for species σ ($\sigma = e, i$), respectively, and $\mathbf{i}_y$ is the unit vector in the y direction. Eqs. (1) are derived as follows. Suppose that the particles have a Maxwellian velocity distribution and that the guiding centers are located in $x > 0$ with a constant density $\langle n_\sigma \rangle$. (In $x < 0$, there are no guiding centers.) Then the density $n_{0\sigma}(x)$ and the macroscopic velocity $\mathbf{v}_{0\sigma}(x)$ calculated from the real particle positions and velocities are given by

$$n_{0\sigma}(x) = \frac{1}{2}\langle n_\sigma \rangle \left[1 + \Phi\left(\frac{x}{\sqrt{2}\,\rho_\sigma}\right)\right], \tag{2a}$$

$$\mathbf{v}_{0\sigma}(x) = \frac{1}{\sqrt{2\pi}} \operatorname{sgn}(q_\sigma)\, v_{t\sigma} \exp\left(-\frac{x^2}{2\rho_\sigma^{\,2}}\right) \mathbf{i}_y , \tag{2b}$$

where $n_{0\sigma}(x)$ is expressed in terms of the error function

$$\Phi(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\eta d\xi \exp(-\xi^2). \tag{2c}$$

In LOM, $n_{0\sigma}(x)$ and $\mathbf{v}_{0\sigma}(x)$ in the negative x region are mirror-reflected back to the positive x region. Then, we obtain Eqs. (1) from the following equations:

$$n_\sigma^{\mathrm{LOM}}(x) = n_{0\sigma}(x) + n_{0\sigma}(-x), \tag{3a}$$

$$\mathbf{v}_\sigma^{\mathrm{LOM}}(x) = \mathbf{v}_{0\sigma}(x) + \mathbf{v}_{0\sigma}(-x), \tag{3b}$$

where x is positive.

The physical origin of the surface current (macroscopic velocity) is as follows. In the narrow region near the boundary there are two types of particles. The first consists of particles coming from the wall and moving in a inversed magnetic field. The second consists of particles coming from the inner part of the system. Both types of particles traverse this region in the same direction as illustrated in Fig. 1(b). This results in a macroscopic current along the wall.

It is to be noted that this current quite resembles the diamagnetic current due to the density gradient. Both are produced by the finiteness of the Larmor radius. Then, the current along the wall can excite a drift-wave like instability if there is a small angle θ between the z-axis and the magnetic field (tilted B) (i.e., if the particle motions parallel to the magnetic field are included in the code). The origin of the numerical instability observed by Lee and Okuda using the "reflection scheme" alone, must have been this kinetic instability.

In order to confirm the above prediction and to know the evolution of the instability in detail, we made a simulation with "reflection scheme" in LOM. A simulation model is a 2-1/2 dimensional electrostatic dipole expansion code [11, 12] with a static magnetic field slightly tilted from the z direction in the y direction. Finite-sized particles [13, 14] with Gaussian-shaped charge distributions are used. The schematic diagram of the system is shown in Fig. 2. The plasma is uniform both in the x and the

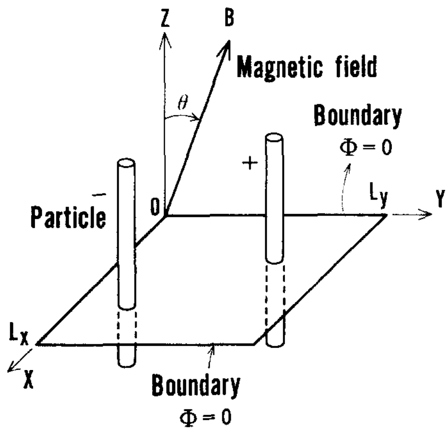

FIG. 2. Sketch of the bounded plasma model in a magnetic field. The cases of the $\theta = 0°$ and $\theta \neq 0°$ correspond to the 2 and 2−1/2 dimensional codes, respectively. Periodic boundary condition is used in the y direction.

y directions and is bounded by the conducting walls at $x = 0$ and $x = L_x$ where the electrostatic potentials are zero. Periodic boundary condition is used in the y direction; particles leaving a boundary at $y = 0$ ($y = L_y$) are reintroduced at the opposite boundary at $y = L_y$ ($y = 0$). The "reflection scheme" in LOM is used at the left wall ($x = 0$). On the other hand, another method (Method I) is used at the right wall ($x = L_x$). Method I is stable and will be discussed in Sec. 3. It will be shown that the plasma is unstable near the left wall and stable near the right wall. Initially the guiding centers of the particles are uniformly loaded with Maxwellian velocity distributions. No quiet start technique [15] is used. The parameters are

> System size, $L_x \times L_y = 32 \times 32$; Number of particles, $N_e = N_i = 8192$; Particle size, $a_x = a_y = 1.5$; Finite time step, $\Delta t = 0.4$; Mass ratio, $m_i/m_e = 25$; Temperature ratio, $T_i/T_e = 0.25$; Electron Debye length, $\lambda_{De} = 1.41$; Electron Larmor radius, $\rho_e = 1.0$; Ion Larmor radius, $\rho_i = 2.5$; Electron cyclotron frequency, $\omega_{ce} = 1.41$; Angle between the magnetic field and the z-axis, $\theta = 1.5°$.

Here lengths and times are normalized by the grid spacing Δ and the inverse of the electron plasma frequency ω_{pe}, respectively.

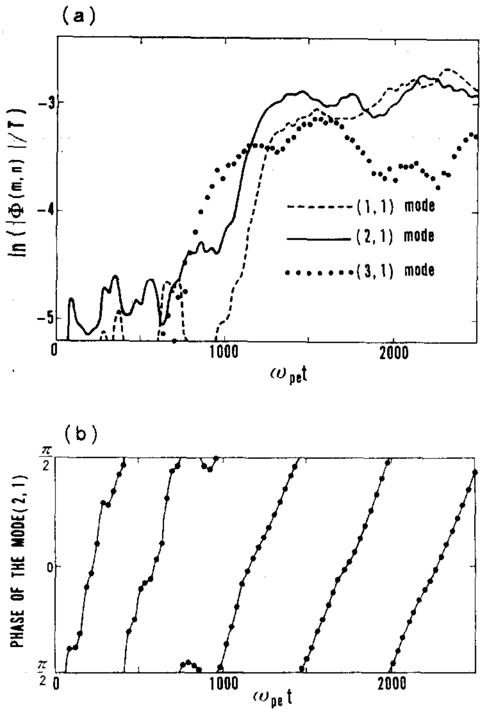

FIG. 3. (a) Growth of Fourier modes of the electrostatic potential. (b) Phase of the mode $(m, n) = (2, 1)$.

Now, let us look at the gross behavior of the surface instability near the left wall. Fig. 3 shows the time dependence of the Fourier components of the electrostatic potential, $(m, n) = (1, 1)$, $(2, 1)$ and $(3, 1)$, where m and n correspond to the wave numbers, $k_x = m\pi/L_x$ and $k_y = 2n\pi/L_y$, respectively. These modes grow above the thermal noise at $t \simeq 800$, exponentiate linearly until $t \simeq 1250$ and saturate at $t \simeq 1400$. Then the amplitudes of the modes keep almost the same values at the saturation stage. No instability is observed for $n = 2$ and higher modes. The observed growth rate is $\gamma = 0.0054$. Fig. 3(b) shows the time dependence of the phase of the mode $(m, n) = (2, 1)$. Excellent coherency is observed after $t \simeq 1000$ and the measured frequency from this is $\omega = 0.0052$. The direction of the phase velocity is that of the macroscopic electron velocity near the wall. This is confirmed by the other simulation using the "reflection scheme" in LOM at the right wall (at the left wall, Method I is used), in which the direction of the phase velocity is changed corresponding to the change of the direction of the surface current.

The spatial structures of the $n = 1$ mode are shown in Fig. 4. The growth of the instability at the LOM boundary ($x = 0$) is clearly observed. The width of the perturbed region is about the ion Larmor diameter at the initial and the middle stage of the linear growth ($t = 880$ and $t = 1040$). It begins to spread in the final stage of the linear growth ($t = 1280$) and it is more than several times the ion Larmor radius in the saturation stage ($t = 1920$). This spreading is critical because the typical size of a simulation plasma transverse to the magnetic field is usually $20 \sim 30\rho_i$ and there are two boundary walls.

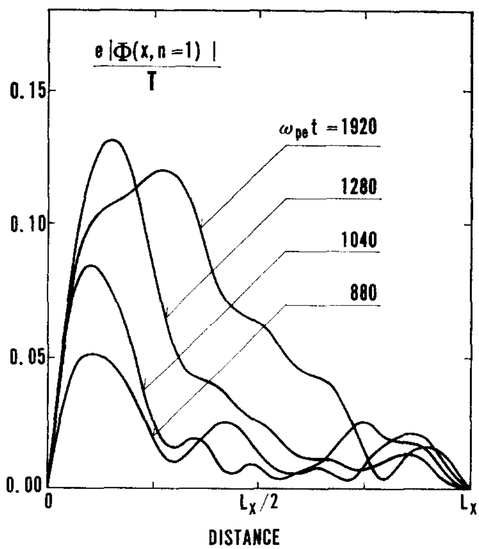

FIG. 4. Mode structure for $n = 1$ mode. The growth of the mode is shown in conjunction with the spreading of the width of the perturbed region. Lee and Okuda's 'reflection scheme' is used at the left wall, while at the right wall Method I is used.

Fig. 5 shows the spatial structures of the macroscopic currents due to the ions. In Fig. 5a which corresponds to the initial time of the simulation, there is a current near the left wall due to a special kind of the treatment of the particles by LOM. Its spatial dependence on x agrees quite well with the prediction of Eqs. (1). At the initial stage of the linear growth (Fig. 5b), the surface current is perturbed in the y direction due to the $E \times B$ drift by the $n = 1$ surface mode. The width of the current layer is also about a ion Larmor diameter. At the final stage of the linear growth (Fig. 5c) the current is considerably modulated by the $n = 1$ mode. Its width is about four times as large as the ion Larmor radius. The change in the current is due to the

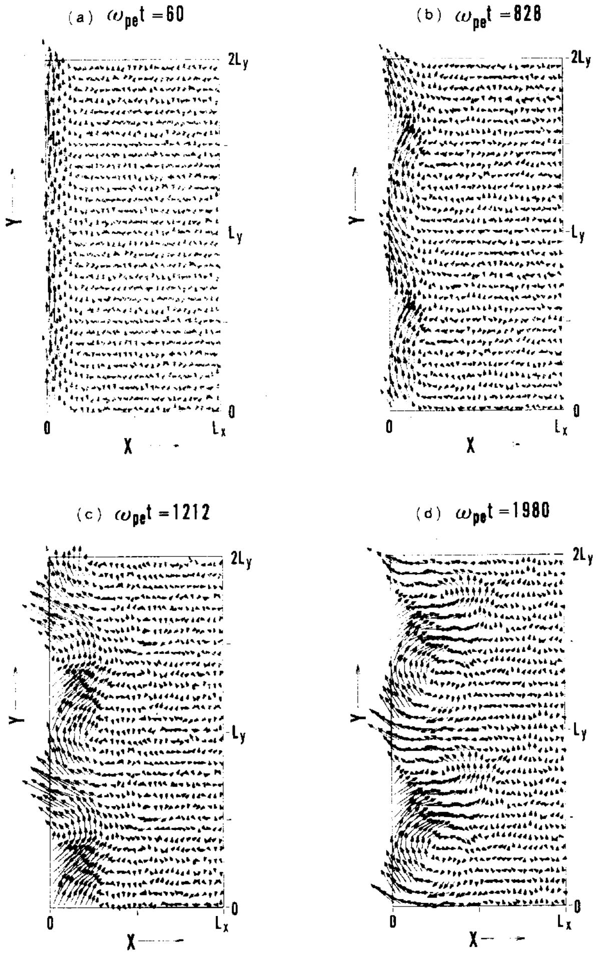

FIG. 5. Spatial structures of the macroscopic current due to the ions. The lengths of the arrows are proportional to the magnitudes of the currents. One fourth of the L_x is equal to the magnitude $n_0 v_{ti}$. Each figure corresponds to (a) the initial time of the simulation, (b) the time when the instability sets in, (c) the final stage of the linear growth and (d) the saturation stage of the instability.

nonlinear transport of the particles via $E \times B$ drift of the surface mode. The averaged x components of the current is directed to the left wall, and this generates the large density gradient near the boundary. In the saturation stage (Fig. 5d) the width of the current layer is more than several times ρ_i. This width agrees quite well with that of the region where $n = 1$ potential mode is dominantly excited (Fig. 4). The form of the current layer is vortex like in this stage, and the initial current profile is completely wiped out.

Electron and ion temperatures which are parallel and perpendicular to the magnetic field are also observed. The electron parallel temperature near the left wall decreases, which indicates that the inverse Landau damping of the electrons is the origin of the instability.

From all the results described above, we know that this instability firstly observed by Lee and Okuda, closely resembles the drift-wave instability [8, 10]. This numerical instability is closely related to the electron motion along the magnetic field and the surface current of the electrons. When the 2 dimensional code is used, this instability does not occur. Moreover, when the guiding center approximation is used to push the electrons (such case is also discussed in Ref. [5]), the instability would not occur because electron surface current is eliminated. Also, the "smoothing scheme" [5] can eliminate the instability by retarding the growth of the surface wave.

III. Modifications of the Boundary Conditions

In order to suppress the numerical instability, Lee and Okuda have proposed the "smoothing scheme" to be used together with the "reflection scheme." Another way to face the boundary phenomena is to find methods of treating particles which are not unstable in themselves. For this purpose, it is necessary to find methods which introduce no macroscopic current as well as the density gradient. Here we show such boundary conditions. The configuration is the same as the one described in Sec. II.

A. *Method* I

One of the method is to reflect to a particle with the reversed velocity (Method I, see Fig. 1c) at the wall

$$v_x \to -v_x, \tag{4a}$$

$$v_y \to -v_y, \tag{4b}$$

where the coordinate system perpendicular to the magnetic field is assumed (the case of $\theta = 0°$ in Fig. 2). This method produces neither a density gradient nor a macroscopic current as seen from the following modification of Eqs. (3);

$$n_\sigma(x) = n_{0\sigma}(x) + n_{0\sigma}(-x) = \langle n_\sigma \rangle, \tag{5a}$$

$$\mathbf{v}_\sigma(x) = \mathbf{v}_{0\sigma}(x) - \mathbf{v}_{0\sigma}(-x) = \mathbf{0}. \tag{5b}$$

Here, it is assumed that the particles have been loaded as to satisfy Eqs. (5); an actual method of initially loading particles is described in the following subsection concerning Method II. When one uses a 2-1/2 dimensional code, one must reflect only the components of the velocity perpendicular to the magnetic field;

$$v_x \to -v_x\,, \tag{6a}$$

$$v_y \to -v_y \cos 2\theta + v_z \sin 2\theta, \tag{6b}$$

$$v_z \to v_y \sin 2\theta + v_z \cos 2\theta. \tag{6c}$$

Since the time is discretized in the simulations, one should determine the new position of the particle with linear interpolations

$$x \to 2x_0 - x, \tag{7a}$$

$$y \to r(y_- + v_y{}^N \Delta t) + (1 - r)\,y, \tag{7b}$$

$$r = (x - x_0)/(x - x_-), \tag{7c}$$

where x_0 represents the position of the wall (here, $x = 0$); $v_y{}^N$ means the newly determined velocity; x_- and y_- show the positions of the particle at the last time step (i.e., $x_- = x - v_x \Delta t$, $y_- = y - v_y \Delta t$). Here we are assuming the leap-frog-scheme for the particle pushing algorithm. Eqs. (7) can be used both for the 2 and 2-1/2 dimensional codes.

One may think that the guiding center shifts at the wall inherent to Method I would dominate the physics. However the current produced by the guiding center shifts of the particles is completely cancelled out by the current produced by the particles just missing the wall. Therefore there is no net macroscopic current along the wall.

Method I has been used at the right wall ($x = L_x$) of the system in the simulation which is presented in the previous section. Fig. 5a shows that initially there is no macroscopic current near the right wall. From Fig. 4 and Figs. 5b,d, it is shown that Method I is stable and that it does not affect the phenomenon in the interior of the plasma. In fact the surface instability excited at the left wall is successfully simulated without any interference from the right wall. This method has been used quite satisfactorily in the simulation studies of the drift-wave instabilities caused by the temperature gradient [16].

B. *Method* II

If we could choose the initial condition correctly with the reflecting boundary condition, the net surface current would disappear as in the case of Method I. One such set of initial conditions is as follows: first the guiding centers of the particles are uniformly loaded in the system and the real velocities are assigned to these particles; secondly, the real positions of the particles are determined, some of which are outside the boundary; thirdly, the positions and the velocities of the particles outside the system are changed as

$$x \to 2x_0 - x, \tag{8a}$$

$$v_x \to v_x, \tag{8b}$$

$$v_y \to -v_y, \tag{8c}$$

where new guiding centers of these particles are outside the boundary. (See Fig. 1d.) In the 2-1/2 dimensional code, v_y and v_z should be changed following (6b) and (6c), while x and v_x should be changed following Eqs. (8a) and (8b).

Method II is the combination of this modified uniform guiding center loading and the reflecting boundary condition. Method I also uses this initial condition. Let us show that Method I and II give the same macroscopic quantities ($n_o(x)$, $\mathbf{v}_o(x)$, etc.) if only the unperturbed orbits of the particles are taken into consideration. Suppose that in Fig. 6 particles which are to move along the orbits a, b in Method I and c, d in Method II, are uniformly loaded on these orbits (viz., the number density of the particles on these orbits are the same everywhere). Then these systems are stationary and one cannot distinguish between these systems from the macroscopic point of view.

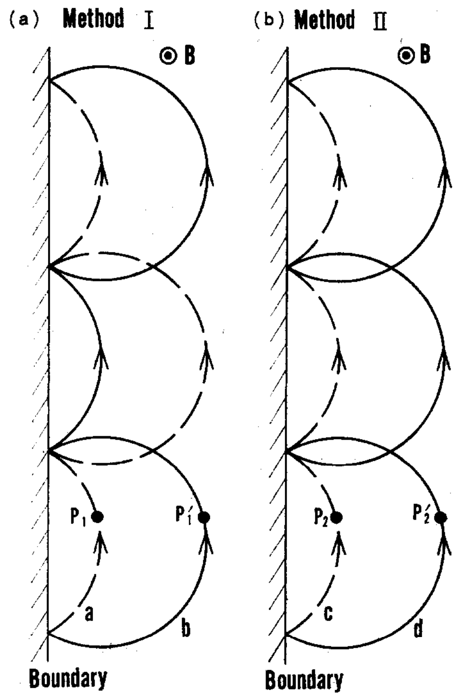

FIG. 6. Illustration that (a) Method I and (b) Method II are same macroscopically. Dashed lines (a, c) and Solid lines (b, d) represent the unperturbed orbits of reflected particles. It is supposed that particles are located uniformly on these orbits.

(From the microscopic point of view, for example, a particle $p_1(p_1')$ and a particle $p_2(p_2')$ which are located at the same position in the each system move differently according to the method of reflection.) If the initial particle distributions are determined from the superpositions of these particle distributions, such systems are also same macroscopically. The initial condition described here satisfies this condition. One can then conclude that Method I and II are same macroscopically and Method II also does not produce the unphysical surface phenomenon.

The initial condition presented here is not the unique one and one will be able to make many modifications, if one want to reduce the statistical fluctuations.

The simulation is made with Method II with the same system and parameters shown in Section 2. It is found that there is neither a density gradient nor a surface current and that there is no surface instability. Additional simulation is done with another incorrect initial conditions where the velocities and the positions of the particles outside the system in the uniform guiding center loading are changed as $v_x \to -v_x$, $x \to 2x_0 - x$. The reflecting boundary condition is used. As pointed out by Lee and Okuda [5] and described in Section 2, it is found that there is a large density gradient and a macroscopic current. These effects are produced by the change in the probabilities of the particle distributions on orbit c and d in Fig. 6b due to the incorrect initial conditions. All the guiding centers of the particles are inside the system. In addition the dominant surface current is not due to the guiding center shifts along the wall but due to the particles gyrating in the system near the wall (viz., due to the diamagnetic current).

Method II makes use of the modified uniform guiding center loading as the initial condition. If the real positions of the particles are uniformly loaded with the reflecting boundary condition, such a system is also free from the density gradient and the macroscopic current near the wall. However uniform guiding center loading is preferable in many cases. It has less fluctuations if the magnetic field is relatively strong. When the plasma where the inhomogeneity exists only in the central region away from the wall is treated, slight modification of the uniform guiding center loading can be used for the initial condition.

In connection with Method II let us consider the fully ionized plasma confined by the perfectly reflecting walls. It is well known that even in a fully electromagnetic system, such a plasma has no magnetic effect if it is in thermodynamic equilibrium. This is because the motion of the particles that are reflected by the walls represents a current, and the magnetic moment of this current exactly cancels the magnetic moment of particles gyrating in the interior [17, 18]. Although the electrostatic code used here does not include the magnetic effect, what is important is that the code can realize the thermodynamic equilibrium state. Method II realizes this state and produces no unphysical surface effect near the boundary. Moreover even if the code includes magnetic effects, Method II can be used without modifications.

C. *Random Reflection Method*

There is another set of boundary conditions where particles are reflected with the given velocity distribution. In the unmagnetized plasma, particles are reintroduced with a half-Maxwellian velocity distribution [7]. However in the magnetized plasma we have the velocity distributions as

$$f_\sigma(v_x) = \sigma_j(|v_x|/v_{t\sigma}^2)\exp(-v_x^2/2v_{t\sigma}^2), \tag{9a}$$

$$f_\sigma(v_y) = (1/\sqrt{2\pi}\, v_{t\sigma})\exp(-v_y^2/2v_{t\sigma}^2), \tag{9b}$$

where

$$\begin{aligned} j &= R, L, \\ \sigma_R = -1, &\qquad \sigma_L = 1, \end{aligned} \tag{9c}$$

where subscripts R and L represent the left and right walls, respectively. Eqs. (6) are obtained by calculating the velocity distribution of particles which cross the wall at $x = x_0$ in the $+x$ $(-x)$ direction assuming a homogeneous plasma whose guiding centers are located uniformly in the whole space with a Maxwellian velocity distribution. Because the energetic particles have larger Larmor radius than the cold particles, larger number of energetic particles can cross the wall at $x = x_0$. This gives rise to a change in a velocity distribution described above.

One must determine the new position of a reflected particle with the linear interpolations;

$$x \to x_0 + r v_x^N \Delta t, \tag{10a}$$

$$y \to r(y_- + v_y^N \Delta t) + (1 - r)\, y, \tag{10b}$$

where v_x^N and v_y^N are the new velocities according to Eqs. (9) and r is given by Eq. (7c). It is straightforward to extend Eqs. (9) and (10) to the 2-1/2 dimensional code. We also checked this method and found that it produces neither density gradient nor surface current. When the linear interpolations in Eqs. (10) are not used, however, the density gradient and the macroscopic current become large as the time steps increase.

D. *Nevins and Gerver's Method*

Let us see the method proposed by Nevins and Gerver (we call this NGM) [6] to compare it with methods described earlier. In NGM, it is assumed that the plasma has inverse symmetry around the point ($x = 0$, $y = 0$). (The region to be simulated is $0 < x < L_x$ and $-L_y/2 < y < L_y/2$.) Then a particle going out of the $x = 0$ boundary at $y = y_1$ is reintroduced at $y = -y_1$ and $x = 0$, with its velocity (x, y, z components) reversed. This holds both for the 2 and 2-1/2 dimensional codes. The advantage of NGM is that it is equivalent to having no wall at all at $x = 0$. NGM will have no instability near the wall because it produces neither a density gradient nor a macroscopic current. Boundary condition for the field is also restricted by the inverse symmetry condition. The upper ($y > 0$) and the lower ($y < 0$) domain are connected directly at the boundary. Then if a strong instability is excited in the interior of the plasma, there may be strong coupling of the upper and the lower domains through the boundary. (NGM is developed for the drift wave simulations.) However, if the density gradient are restricted to the region away from the wall at $x = 0$ (it is the case employed by Nevins and Gerver [6]), NGM would have no problem becaues the instability is also restricted to that region.

Compared with LOM, Method I, Method II and the random reflection method are simple and straightforward with no mathematical or physical complications. These are better than LOM because these do not generate the surface current. These are also better than the usual reflecting boundary condition, because these do not produce a density gradient which introduces a large electric field and diamagnetic currents near the boundary.

Compared with NGM, our methods seem to be the results of the different approach to the same problem. The former is the results of the endeavor to eliminate the walls and the latter is that to make good use of the walls. Both would be same for the phenomena occuring only in the interior of the system away from the walls.

V. Discussions and Conclusions

We have studied different treatments of particles at the boundary wall in a magnetized simulation plasma. Especially, the "reflection scheme" proposed by Lee and Okuda is investigated in detail. It is found that the numerical instability caused by this scheme (the existence of which was first observed by Lee and Okuda [5]) is the kinetic instability coming from the macroscopic current along the wall. The evolution of the instability was quite akin to that of the drift-waves driven by the density gradient. The problem associated with this instability has been remedied by the "smoothing scheme" of the potential fluctuations [5].

We have described three methods (Methods I and II and the random reflection method) to treat the particles at the walls. They are simple and straightforward, and introduce neither a density gradient nor a surface current, and hence avoid any surface instability. When one uses a bounded plasma model to simulate the phenomena in the interior of the system away from the walls, Method I as well as Method II would be appropriate. However when one wants to simulate the phenomena occuring dominantly near the boundary, Method II might be more realistic because the reflecting boundary used in the method has the physical interpretation as the elastic collisions between the particles and the plane walls. The random reflection method can be used for aims such as fixing the temperature at the walls.

In this paper, we have presented these methods so as to work very well for homogeneous plasmas. These methods can also be used for a system where the inhomogeneity exists only in the central region away from the walls. Now, let us consider a case where the sharp inhomogeneity exists next to the wall. For simplicity we consider only Method I and Method II. In this case, it is necessary that the method of initially loading particles realizes the given equilibrium distribution and that the method of treating particles does not change this distribution. The extension of the methods to satisfy these conditions is straightforward if the out-of-bounds particles determined by the guiding center loading are replaced into the system following the same method described concerning Method II. However, the charge separation remains for this case, because the actual charge densities for different species of particles are not the same. Then, there is a possibility of weak instability (we have not verified this case in actual simulations). The same problem exists for LOM, although it may not so important because of the "smoothing scheme" used together. The new initial loading scheme which does not generate the charge separation is needed. For such a distribution, Method II will be useful because reflecting boundary condition can maintain arbitrary distributions. At this point in time, Lee and Okuda's method is the only method to handle such a problem.

The surface current introduced by some method, distrubs the interior of the plasma only after the time when the instability occurs in the electrostatic code. However in the magnetostatic code [19] this surface current itself can give rise to the magnetic field and plasma will be strongly diamagnetic. The phenomena one wants to simulate will considerably change. Our methods are useful not only for the 2 and 2-1/2 dimensional electrostatic codes verified in this paper, but they will be also useful for the 3 dimensional electrostatic code and the magnetostatic particle simulation code.

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